5-2 Dividing Monomials

Objective: To simplify quotients of monomials and to find the GCF of several monomials.

Vocabulary

Greatest common factor (GCF) of two or more monomials The common factor with the *greatest coefficient* and the *greatest degree* in each variable. For example, $5x^2y$ is the GCF of $10x^2y^2$ and $25x^3y$.

Properties and Rules

Property of Quotients

If a, b, c, and d are real numbers with $b \neq 0$, and $d \neq 0$, then $\dfrac{ac}{bd} = \dfrac{a}{b} \cdot \dfrac{c}{d}$.

For example, $\dfrac{15}{16} = \dfrac{3 \cdot 5}{2 \cdot 8} = \dfrac{3}{2} \cdot \dfrac{5}{8}$

Rule for Simplifying Fractions

If b, c, and d are real numbers with $b \neq 0$ and $d \neq 0$, then $\dfrac{bc}{bd} = \dfrac{c}{d}$.

For example, $\dfrac{15}{18} = \dfrac{3 \cdot 5}{3 \cdot 6} = \dfrac{5}{6}$.

Rule of Exponents for Division

If a is a nonzero real number and m and n are positive integers, then:

If $m > n$: $\dfrac{a^m}{a^n} = a^{m-n}$

If $n > m$: $\dfrac{a^m}{a^n} = \dfrac{1}{a^{n-m}}$

If $m = n$: $\dfrac{a^m}{a^n} = 1$

CAUTION You can divide the numerator and denominator of a fraction only by a nonzero number. In the examples of this lesson, *assume that no denominator equals zero.*

Example 1 Simplify: **a.** $\dfrac{28}{35}$ **b.** $\dfrac{-15xy}{21x}$

Solution **a.** Divide both numerator and denominator by 7. The "cancel" marks show this.

$$\dfrac{28}{35} = \dfrac{4 \cdot \cancel{7}}{5 \cdot \cancel{7}} = \dfrac{4}{5}$$

b. Divide both numerator and denominator by $3x$.

$$\dfrac{-15xy}{21x} = \dfrac{3\cancel{x}(-5y)}{3\cancel{x} \cdot 7} = \dfrac{-5y}{7}, \text{ or } -\dfrac{5y}{7}$$

Example 2 Simplify: **a.** $\dfrac{x^8}{x^3}$ **b.** $\dfrac{x^3}{x^8}$ **c.** $\dfrac{x^2}{x^2}$

Solution **a.** $\dfrac{x^8}{x^3} = x^{8-3} = x^5$ **b.** $\dfrac{x^3}{x^8} = \dfrac{1}{x^{8-3}} = \dfrac{1}{x^5}$ **c.** $\dfrac{x^2}{x^2} = 1$

5-2 Dividing Monomials (continued)

Simplify. Assume that no denominator equals zero.

1. $\dfrac{25}{30}$ **$\dfrac{5}{6}$**   2. $\dfrac{48}{72}$ **$\dfrac{2}{3}$**   3. $\dfrac{54}{72}$ **$\dfrac{3}{4}$**   4. $\dfrac{10^3}{10^6}$ **$\dfrac{1}{10^3}$, or $\dfrac{1}{1000}$**   5. $\dfrac{10^8}{10^5}$ **10^3, or 1000**   6. $\dfrac{10a}{2a}$ **5**

7. $\dfrac{12m}{4m}$ **3**   8. $\dfrac{15 \cdot 10^3}{5 \cdot 10^4}$ **$\dfrac{3}{10}$**   9. $\dfrac{6x^4}{9x^2}$ **$\dfrac{2x^2}{3}$**   10. $\dfrac{4n^6}{20n^4}$ **$\dfrac{n^2}{5}$**   11. $\dfrac{2x^5}{16x^4}$ **$\dfrac{x}{8}$**   12. $\dfrac{12y^3}{3xy^2}$ **$\dfrac{4y}{x}$**

13. $\dfrac{4a^2b}{16ab^2}$ **$\dfrac{a}{4b}$**   14. $\dfrac{-6x^2y^3}{9xy^2}$ **$-\dfrac{2xy}{3}$**   15. $\dfrac{-8a^2b}{-20ab}$ **$\dfrac{2a}{5}$**   16. $\dfrac{-32cd^3}{-24bd^2}$ **$\dfrac{4cd}{3b}$**   17. $\dfrac{-21bc^3}{-14cd^2}$ **$\dfrac{3bc^2}{2d^2}$**

18. $\dfrac{30xz^3}{-35yz^2}$ **$-\dfrac{6xz}{7y}$**   19. $\dfrac{x^2yz^3}{x^3y^3z^3}$ **$\dfrac{1}{xy^2}$**   20. $\dfrac{a^2b^4c}{a^2bc^3}$ **$\dfrac{b^3}{c^2}$**   21. $\dfrac{35a^2b^3c}{25abc}$ **$\dfrac{7ab^2}{5}$**   22. $\dfrac{26x^2yz}{52xyz}$ **$\dfrac{x}{2}$**

Example 3 $\dfrac{(9x)^2}{(3x)^3} = \dfrac{81x^2}{27x^3} = \dfrac{27x^2 \cdot 3}{27x^2 \cdot x} = \dfrac{3}{x}$

Simplify. Assume that no denominator equals zero.

23. $\dfrac{(2x)^3}{2x^3}$ **4**   24. $\dfrac{5m^2}{(5m)^2}$ **$\dfrac{1}{5}$**   25. $\dfrac{(2t^2)^3}{(2t^3)^2}$ **2**   26. $\dfrac{(4a^2)^3}{(4a^3)^2}$ **4**   27. $\dfrac{(3ab)^2}{3a^2b}$ **$3b$**

28. $\dfrac{(2mn)^3}{2m^3n^2}$ **$4n$**   29. $\dfrac{(-z)^6}{(-z)^3}$ **$-z^3$**   30. $\dfrac{(-a)^5}{(-a)^3}$ **a^2**   31. $\dfrac{(-xy)^7}{xy^7}$ **$-x^6$**   32. $\dfrac{(-t^3)^4}{(-t^2)^5}$ **$-t^2$**

Example 4 Find the missing factor. $45x^2y^3z^4 = (3xyz^2)(?)$ **Solution** $\dfrac{45x^2y^3z^4}{3xyz^2} = 15xy^2z^2$

Find the missing factor.

33. $8t^4 = (2t)(?)$ **$4t^3$**   34. $10w^4 = (2w^2)(?)$ **$5w^2$**   35. $6a^3b^5 = (2a^2b^2)(?)$ **$3ab^3$**

36. $15pq^3 = (5pq)(?)$ **$3q^2$**   37. $-28x^2y^4 = (7x^2y)(?)$ **$-4y^3$**   38. $-32a^5b^4 = (-8a)(?)$ **$4a^4b^4$**

Example 5 Find the GCF of $18x^3y$ and $10x^2y^3$.

Solution $\left.\begin{array}{l} 18 = 2 \cdot 3^2 \\ 10 = 2 \cdot 5 \end{array}\right\}$ GCF $= 2$ $\left.\begin{array}{l} x^3y \\ x^2y^3 \end{array}\right\}$ GCF $= x^2y$ $\Bigg\}$ GCF $= 2x^2y$

Find the GCF.

39. $21x^3$, $14x^2$ **$7x^2$**

40. a^3b^2, a^2b^3 **a^2b^2**

41. $6xy^2$, $8x^4y^3$ **$2xy^2$**

42. $18c^2d^3$, $24c^2d$ **$6c^2d$**

43. $35pq^2r$, $25p^3qr^2$ **$5pqr$**

Mixed Review Exercises

Simplify.

1. $\dfrac{1}{4}(-24)$ **-6**   2. $105 \cdot \dfrac{1}{5}$ **21**   3. $378 \div 9$ **42**   4. $4n^3\left(\dfrac{1}{4}n^3\right)$ **n^6**   5. $12 \div \left(-\dfrac{1}{3}\right)$ **-36**   6. $10y \cdot \dfrac{2}{5}y^2$ **$4y^3$**